$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\dot{\boldsymbol{x}}^\top \boldsymbol{m}\dot{\boldsymbol{x}} + V(\boldsymbol{x})\right) = \frac{1}{2}\left(\ddot{\boldsymbol{x}}^\top \boldsymbol{m}\dot{\boldsymbol{x}} + \dot{\boldsymbol{x}}^\top \boldsymbol{m}\ddot{\boldsymbol{x}}\right) + \dot{\boldsymbol{x}}^\top \nabla V(\boldsymbol{x})$$
$$= \dot{\boldsymbol{x}}^\top (\boldsymbol{m}\ddot{\boldsymbol{x}} + \nabla V(\boldsymbol{x})) = \dot{\boldsymbol{x}}^\top \mathbf{0} = 0,$$